

GLEE: Geometric Laplacian Eigenmap Embedding

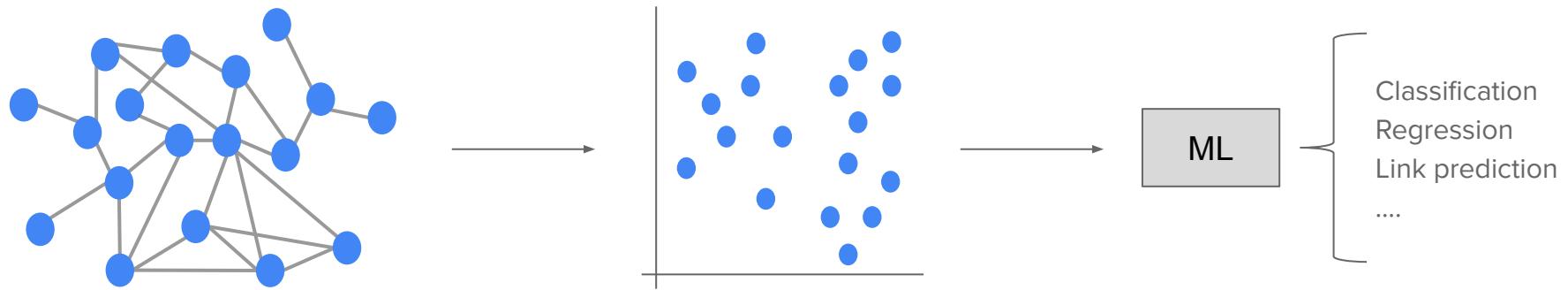
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The problem of embedding



Laplacian Eigenmaps

$$\begin{aligned} \min_Y \operatorname{tr}(Y^t L Y) \\ s.t. \quad Y^T D Y = I \end{aligned}$$

Solution given by the eigenvectors of the normalized Laplacian $\mathbf{D}^{-1}\mathbf{L}$.

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$$\begin{array}{|c|c|c|c|}\hline & & & \\ \hline & & & \\ \hline\end{array}$$

=

$$\begin{array}{|c|c|c|}\hline & & \\ \hline & & \\ \hline\end{array}$$

\times

$$\begin{array}{|c|c|c|}\hline \text{dark green} & & \\ \hline & \text{light green} & \\ \hline & & \text{light green} \\ \hline\end{array}$$

$$\begin{array}{|c|c|c|}\hline \text{dark green} & & \\ \hline & \text{light green} & \\ \hline & & \text{light green} \\ \hline\end{array}$$

\times

$$\begin{array}{|c|c|c|c|}\hline & & & \\ \hline & & & \\ \hline\end{array}$$

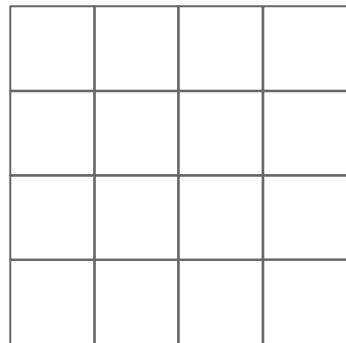
L

P

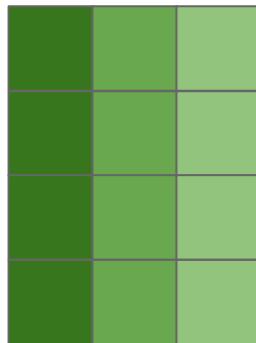
$\sqrt{\Lambda}$ $\sqrt{\Lambda}$

P^T

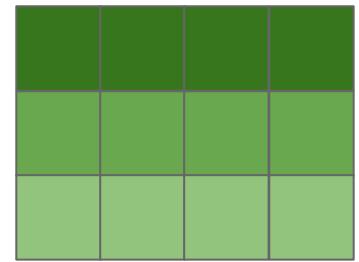
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=



×



L

S

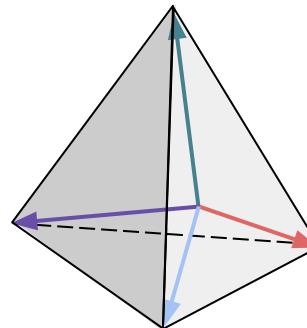
S^T

$$S = P\sqrt{\Lambda}$$

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$$\begin{array}{c|c|c|c} \hline & & & \\ \hline \end{array} = \begin{array}{c|c|c} \hline \text{light blue} & \text{light blue} & \text{light blue} \\ \hline \text{purple} & \text{purple} & \text{purple} \\ \hline \text{teal} & \text{teal} & \text{teal} \\ \hline \text{red} & \text{red} & \text{red} \\ \hline \end{array} \times \begin{array}{c|c|c|c} \hline & & & \\ \hline \end{array}$$

Rows of \mathbf{S} point to the vertices of an
(n-1)-D simplex.

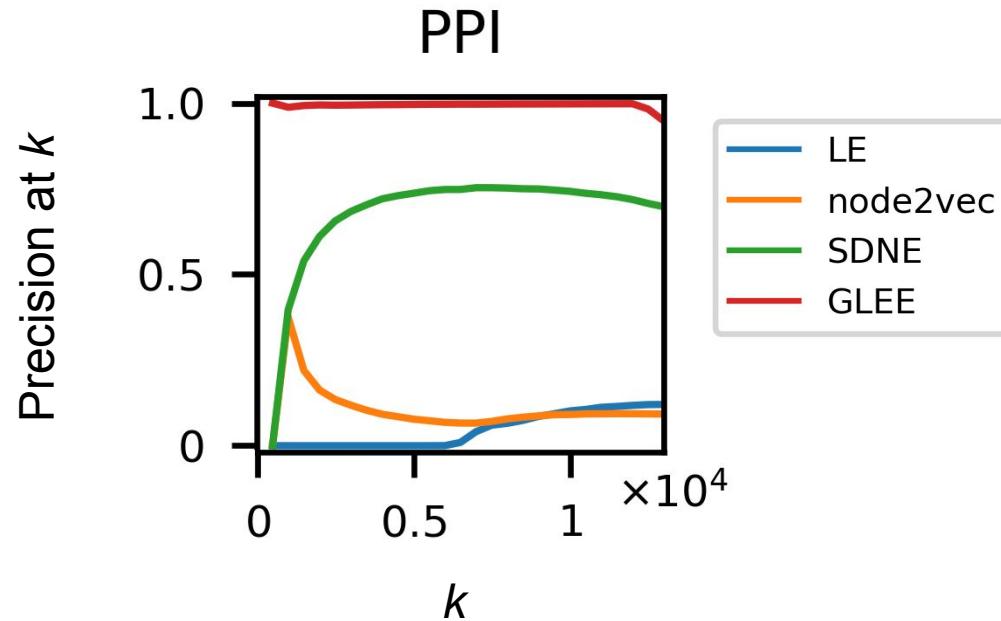


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$$L = i \begin{matrix} d \\ S \end{matrix} \times \begin{matrix} S^T \\ \text{---} \end{matrix}$$

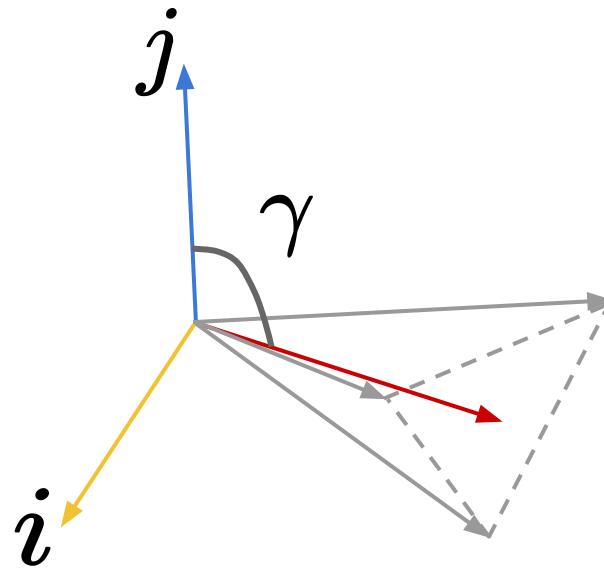
The diagram illustrates the GLEE decomposition of a matrix L . On the left, a large 4x4 grid represents the matrix L . To its right is an equals sign followed by a row vector i , which has a dimension indicator d above it. This vector is composed of four colored segments: blue at the top, purple in the middle, teal at the bottom, and red at the bottom. To the right of i is a multiplication symbol \times . To the right of \times is another row vector, consisting of four empty boxes, with a horizontal line below it indicating its dimension.

Graph Reconstruction



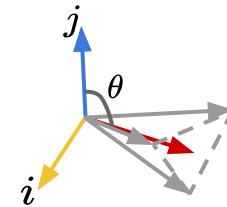
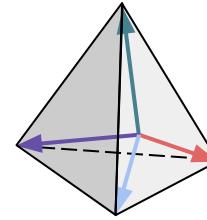
Link Prediction: common neighbors

In many networks (e.g. social networks), the number of common neighbors is an excellent predictor of links because of **triadic closure**.



¡Gracias!

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline \end{array} \times \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline \end{array} \times \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline \end{array}$$



Code: github.com/leotrs/glee

Paper: arxiv.org/abs/1905.09763

Contact: leo@leotrs.com