

# GLEE: Geometric Laplacian Eigenmap Embedding

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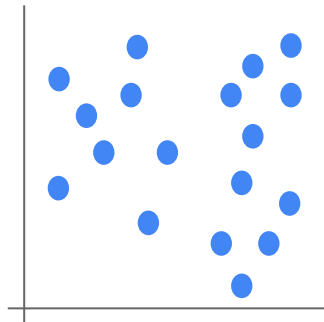
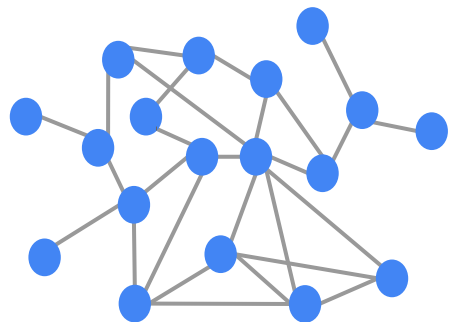
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# The problem of embedding



ML

- Classification
- Regression
- Link prediction
- ....

# Laplacian Eigenmaps

$$\min_Y \operatorname{tr}(Y^t LY)$$
$$s. t. Y^T DY = I$$

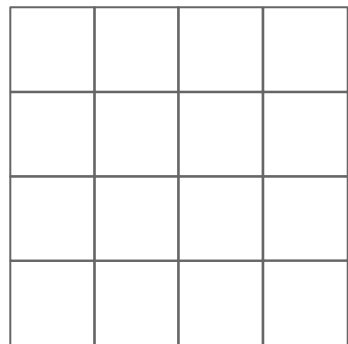
Solution given by the eigenvectors of the normalized Laplacian  $D^{-1}L$ .

# GLEE: Geometric Laplacian Eigenmap Embedding

$$L = P \times \begin{matrix} \times & \begin{matrix} \begin{matrix} \text{Dark Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \end{matrix} & \begin{matrix} \text{Dark Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \end{matrix} \\ \times & \end{matrix} \times P^T$$

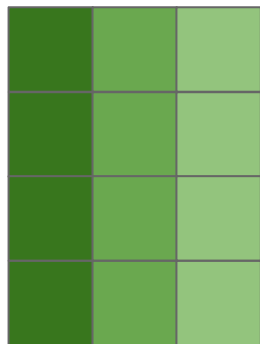
The diagram illustrates the GLEE equation:  $L = P \times \begin{matrix} \times & \begin{matrix} \begin{matrix} \text{Dark Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \end{matrix} & \begin{matrix} \text{Dark Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \\ \text{Light Green} & \text{Light Green} & \text{Light Green} \end{matrix} \\ \times & \end{matrix} \times P^T$ . The matrix  $L$  is a 4x4 grid. The matrix  $P$  is a 4x3 grid. The matrix  $\sqrt{\Lambda} \sqrt{\Lambda}$  is a 3x3 grid with a diagonal of dark green and off-diagonal light green. The matrix  $P^T$  is a 3x4 grid.

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$L$

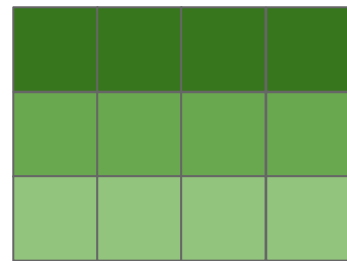
=



$S$

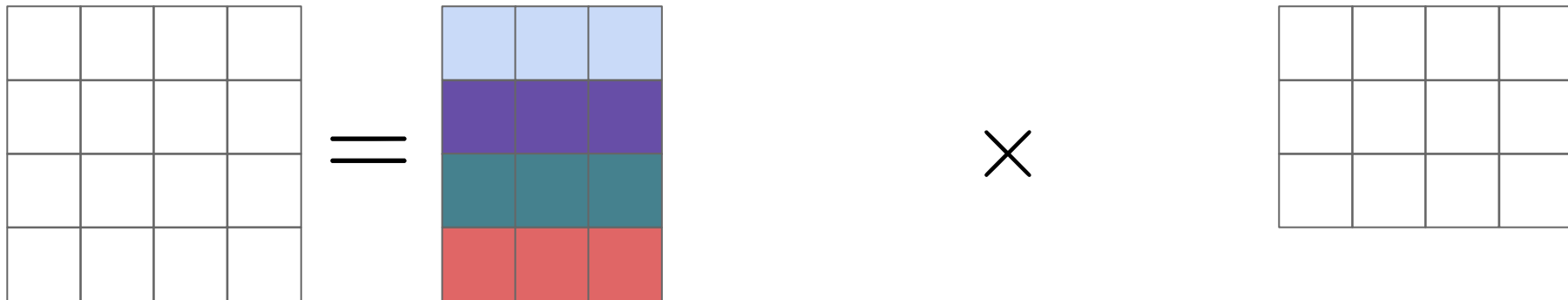
$$S = P\sqrt{\Lambda}$$

×

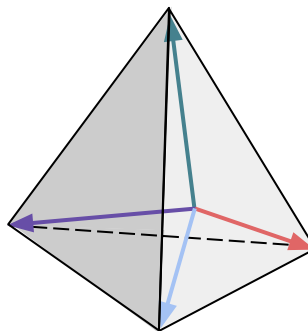


$S^T$

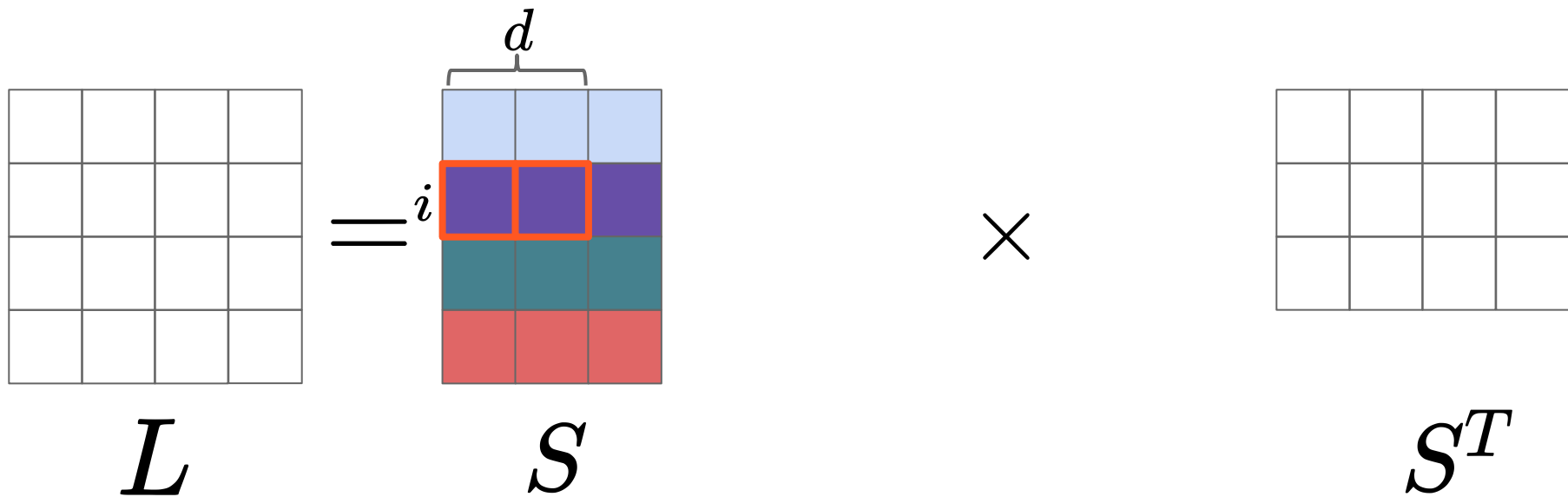
# GLEE: Geometric Laplacian Eigenmap Embedding



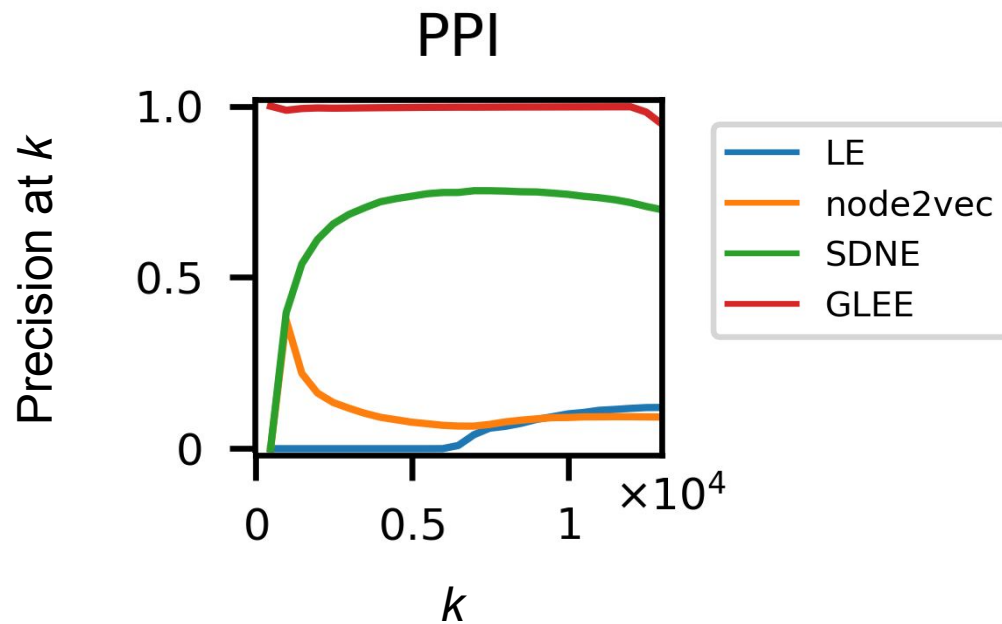
Rows of  $\mathbf{S}$  point to the vertices of an **(n-1)-D simplex**.



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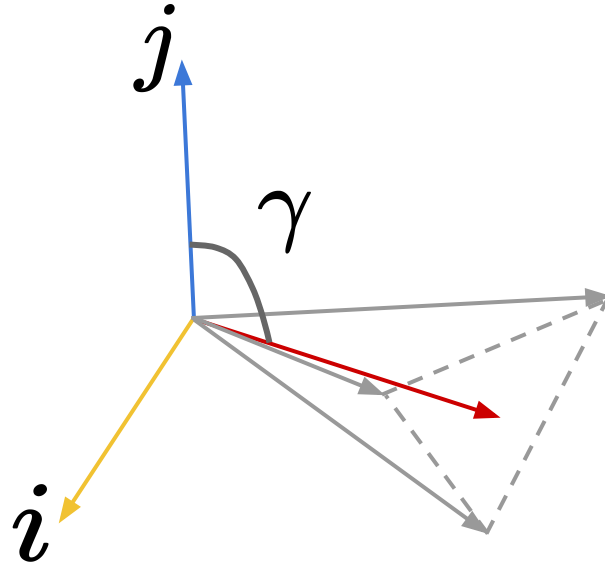
# Graph Reconstruction



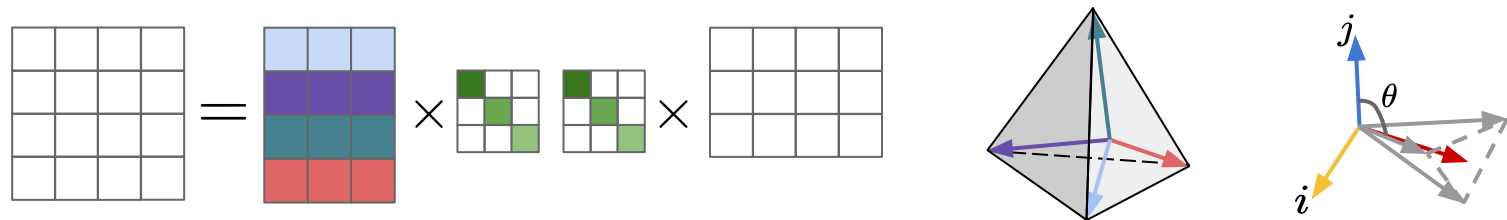


# Link Prediction: common neighbors

In many networks (e.g. social networks), the number of common neighbors is an excellent predictor of links because of **triadic closure**.



# ¡Gracias!



Code: [github.com/leotrs/glee](https://github.com/leotrs/glee)

Paper: [arxiv.org/abs/1905.09763](https://arxiv.org/abs/1905.09763)

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