GLEE: Geometric Laplacian Eigenmap Embedding

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Abstract

- **Graph embedding** builds a low-dimensional representation of a graph.
- Popular in the literature is the *distance-minimization* assumption: if two nodes are close (in the graph), their embeddings must be close (in embedding space).
- We dispose of the distance-minimization assumption. Instead, our new method Geometric Laplacian Eigenmap Embedding (GLEE) builds an embedding with geometric properties by leveraging the so-called simplex geometry of graphs.
- **Benefits of GLEE:**
  - Deterministic and interpretable.
  - Great performance, especially in the case of low clustering.
  - Robust to noise: it can recover graph structure in the presence of a high percentage of noisy edges.

Simplex Geometry and Embedding

1. $L = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
2. $L = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
3. $L = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Graph reconstruction: a classification problem with extreme class imbalance

- **Problem:** find optimal value of $\theta$.

Three solutions:
1. **Constant**
2. **Gaussian Mixtures**
3. **Density Estimation**

**Link Prediction: interpreting the geometry of GLEE**

GLEE (number of common neighbors, CN): $CN(i,j) = \deg(i)\deg(j)N_{ij}^{-1}a_j = \deg(j)CN_{ij}^{-1}a_j$

GLEE-L3 (number of paths of length 3, L3): $L3(i,j) = \deg(i)\deg(j)CN_{ij}^{-1}a_j + \sum_{k\in N(i):k\neq i}||a_k||^2$

Experiments

Conclusions and Future Work

1. GLEE replaces distance-minimization with the direct encoding of graph structure in the geometry of the embedding space.
2. GLEE performs best when the graph has low clustering coefficient, and performance increases as the embedding dimension increases.
3. What other geometric properties of embeddings can we utilize?