A Topologically-Inspired Graph Distance

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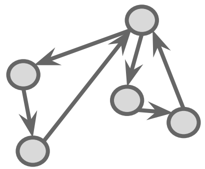
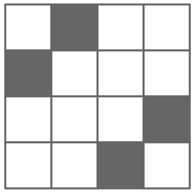
Definitions

- **Backtrack:** traversing an edge twice in succession.

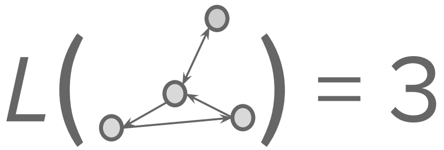
- **Non-backtracking cycle:** a closed walk with no backtracks.



- **Non-backtracking matrix (*B*):** the transition matrix of a random walker that does not do backtracks.



- **Length Spectrum (*L*):** a function that assigns a cycle the length of its “shaved” version.



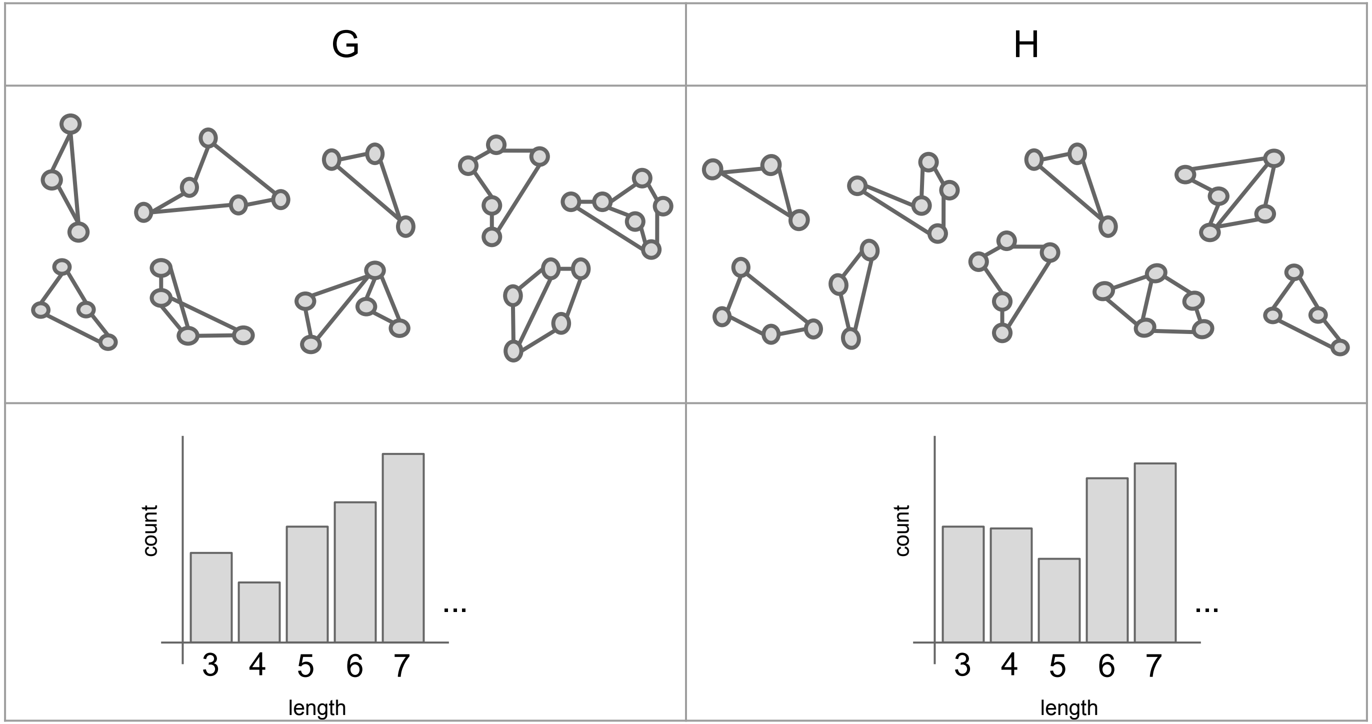
[1] Constantine, David, and Jean-François Lafont. **“Marked Length Rigidity for One-Dimensional Spaces.”** Journal of Topology and Analysis, 2018. doi:10.1142/s1793525319500250.

Using the Length Spectrum

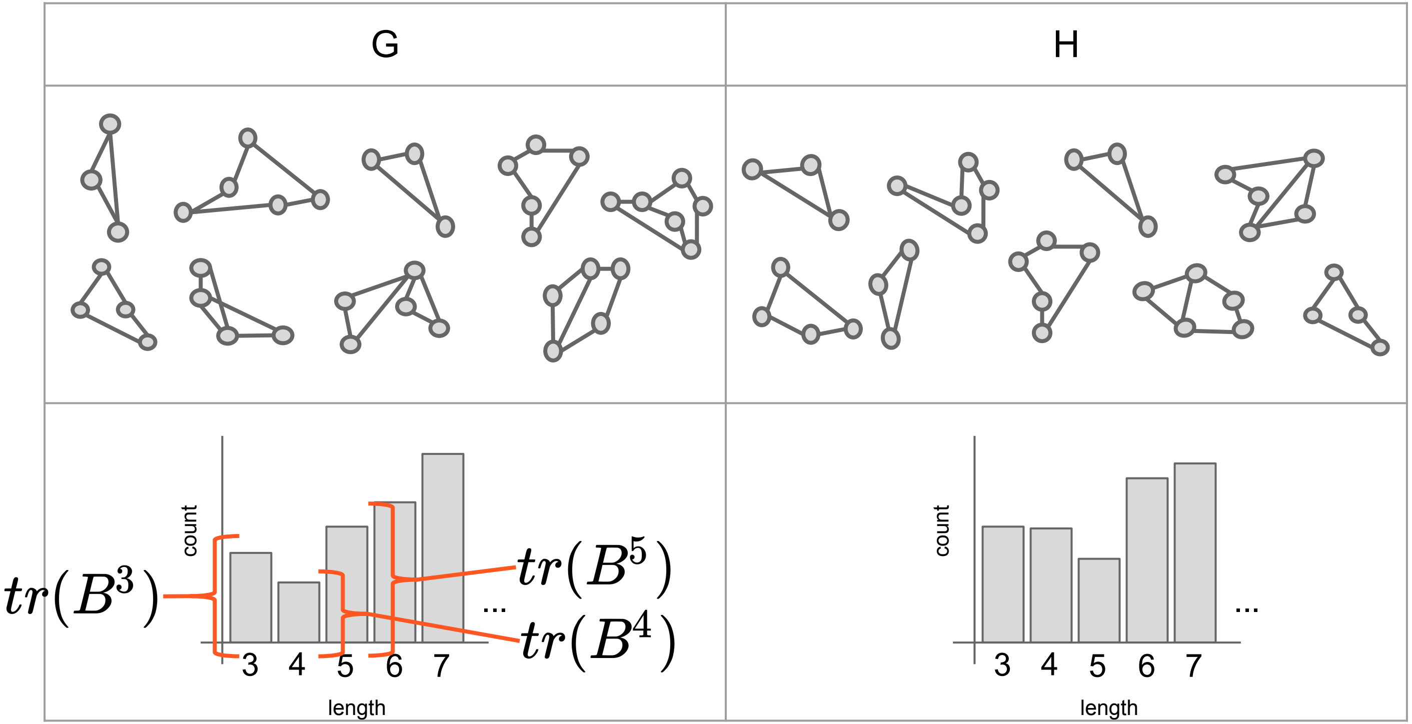
$$d(G, H) = d(\mathcal{L}_G, \mathcal{L}_H)$$

Two assumptions	Two problems
$G \to \mathcal{L}_G$	How to compute?
$d(\mathcal{L}_G, \mathcal{L}_H)$	How to compare?

Using the Length Spectrum



Using the Length Spectrum



Graph Distance

Given two graphs G, H and an integer r, write λ_k, μ_k for the eigenvalues of their corresponding nonbacktracking matrices, $k = 1, 2, \ldots, r$, such that

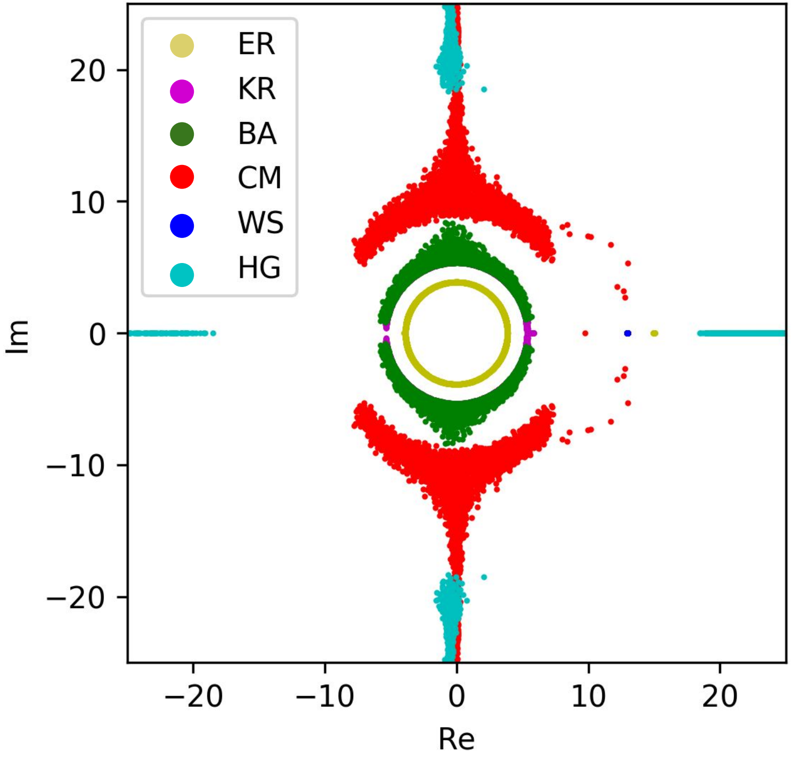
$$|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_r| \qquad |\mu_1| \geq |\mu_2| \geq \ldots \geq |\mu_r|$$

Define their distance by

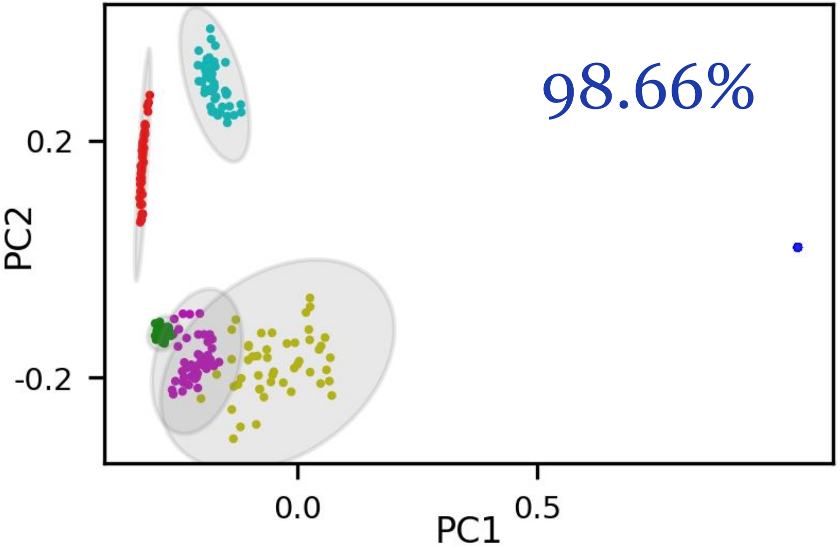
$$d(G, H) = \sqrt{\sum_{k=1}^{r} |\lambda_k - \mu_k|^2}$$

*This defines a pseudo-metric on the set of graphs.

Examples: clustering

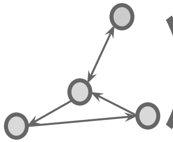


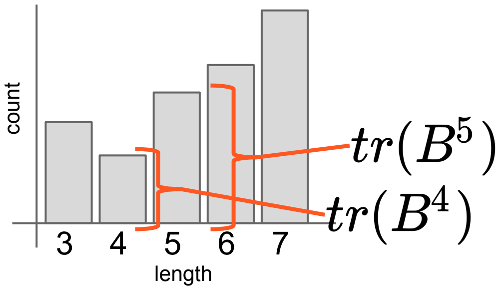
1 dot = 1 eigenvalue

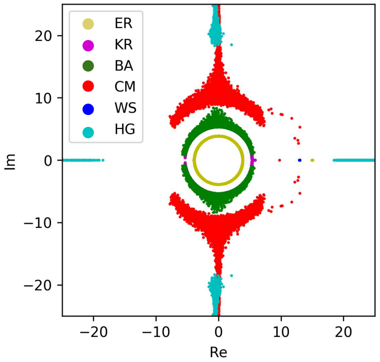


1 dot = 1 graph

Thank You!

$L($  $) = 3$





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Supported by **NSF** CNS-1314603, **NSF** IIS-1741197.