Non-backtracking eigenvalues and X-centrality

Leo Torres
PhD candidate
Network Science Institute, Northeastern University

What happens to the
leading eigenvalue of the
non-backtracking matrix
when a node is removed
from the graph?

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Use this knowledge to define a **centrality measure** for **node immunization**.

Why care about the non-backtracking matrix and its **eigenvalues**?

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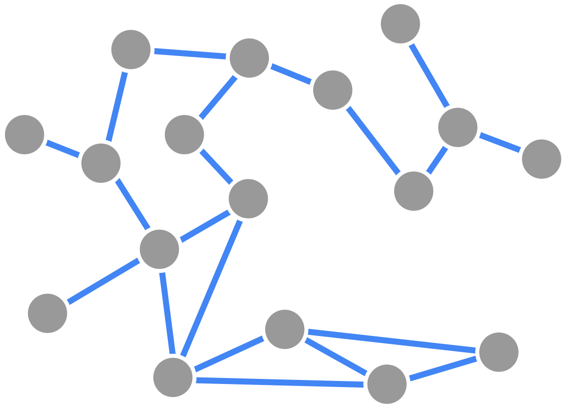
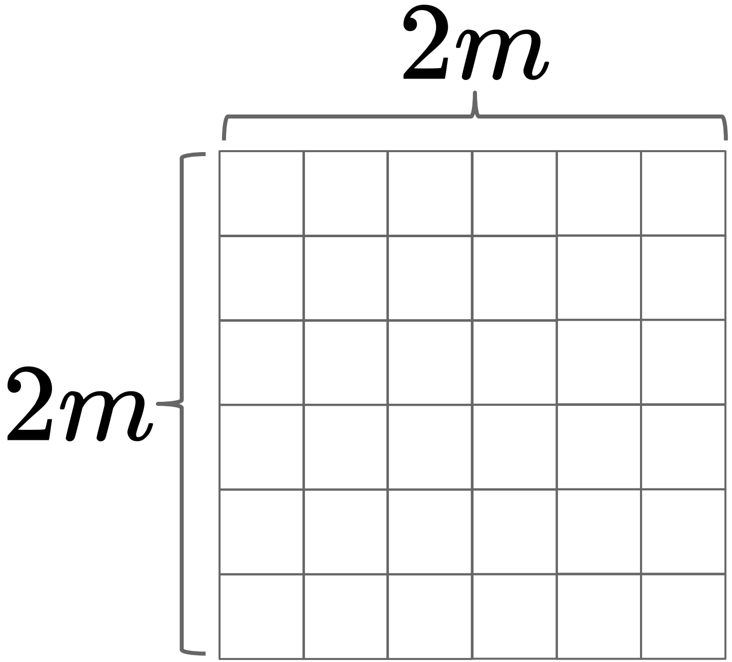
Non-backtracking Matrix

G



$G = (V, E)$
$|E| = m$



B

Non-backtracking Matrix

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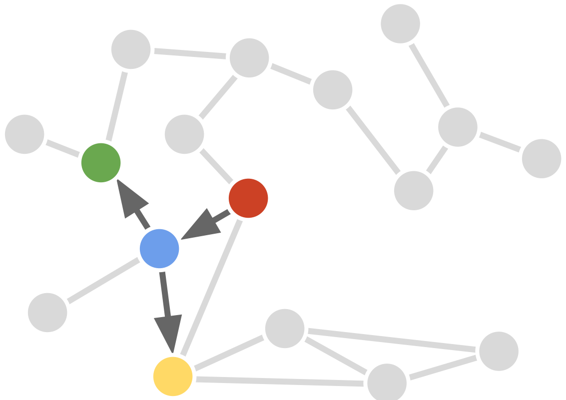

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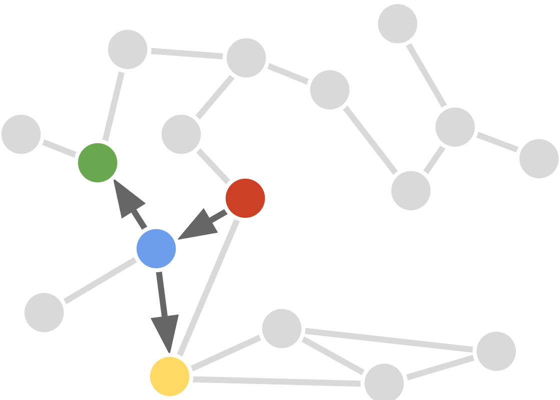

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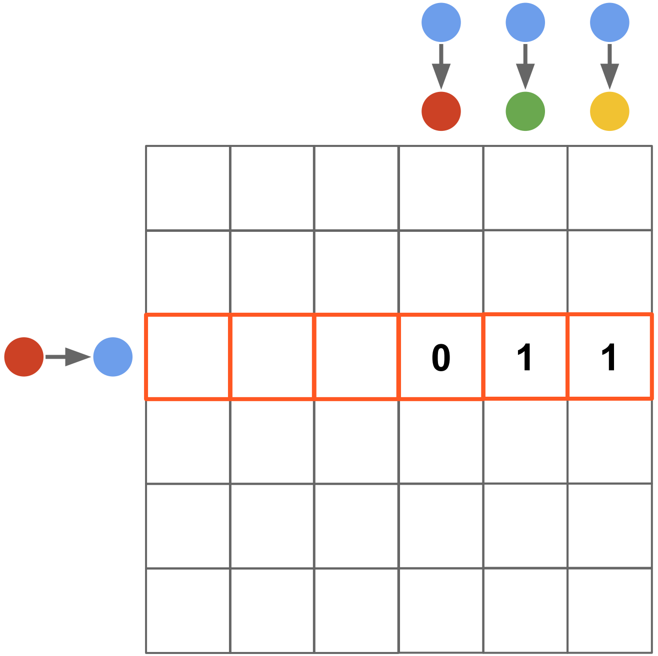

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Non-backtracking eigenvalues

- length spectrum theory
  - Torres, et al. App. Net. Sci. 4.1 (2019): 41.
- community detection
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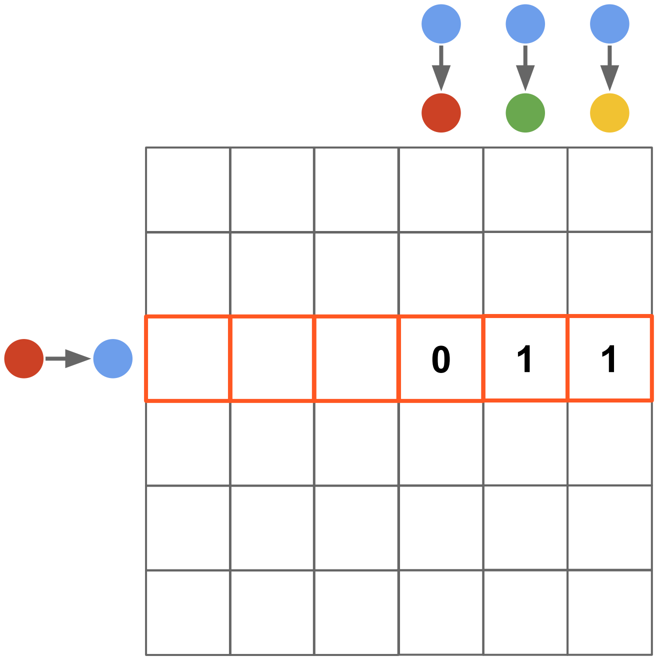

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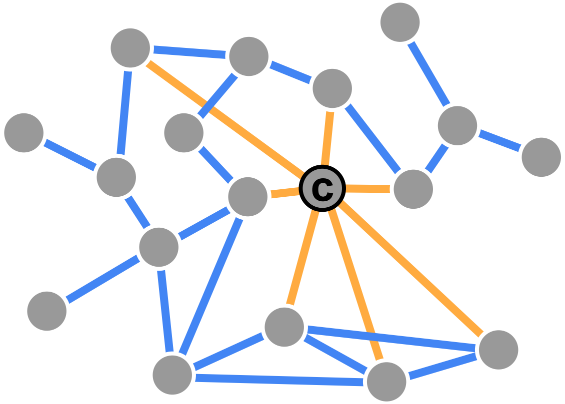
$$\theta \approx 1/\lambda$$

Why care about the non-backtracking matrix and its **eigenvalues**?

What happens to the leading eigenvalue of the **non-backtracking** matrix when a node is removed from the graph?

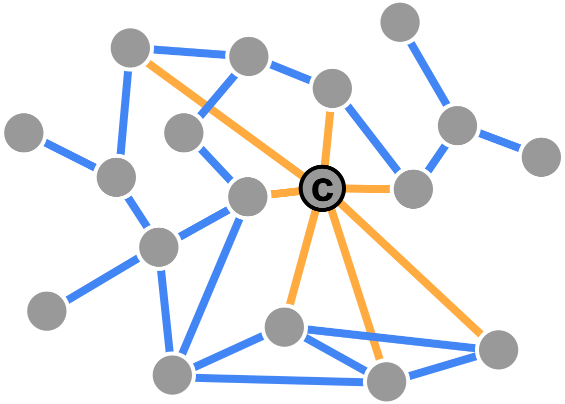
Use this knowledge to define a **centrality measure** for **node immunization**.

Some notation



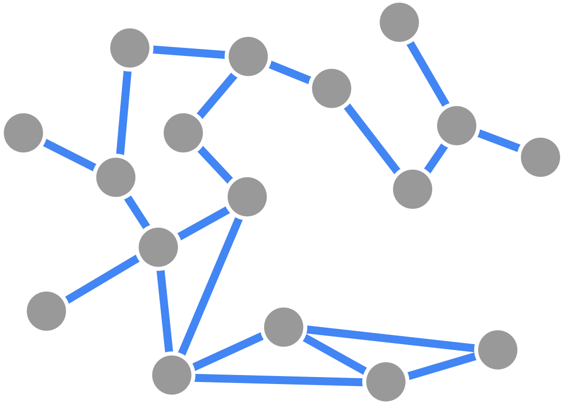
Some notation

G^c



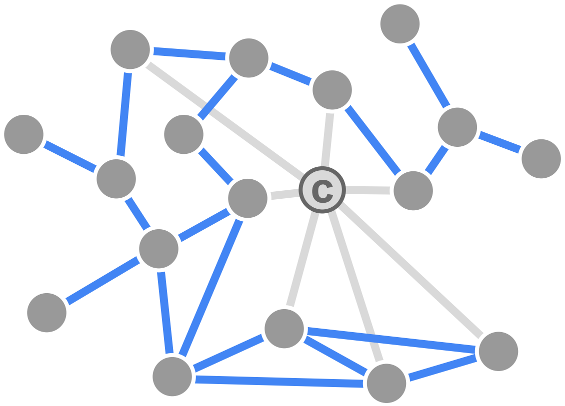
B^c, λ_1^c

G



B, λ_1

Block Matrix



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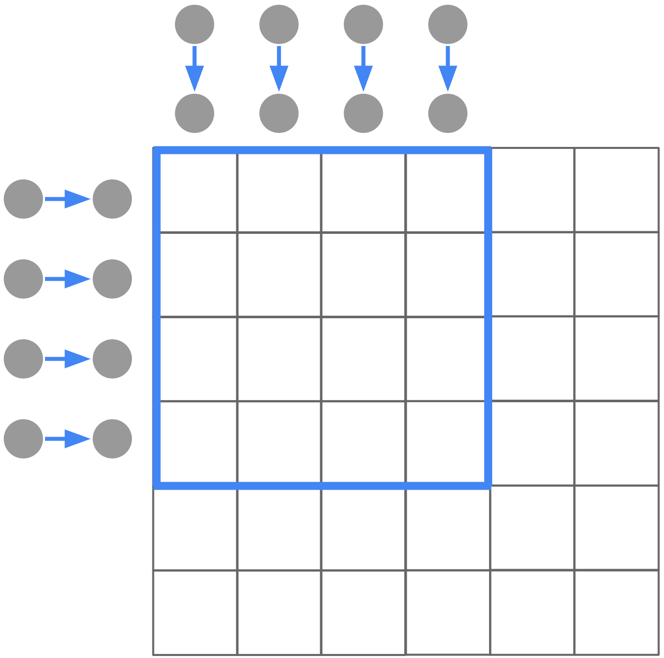

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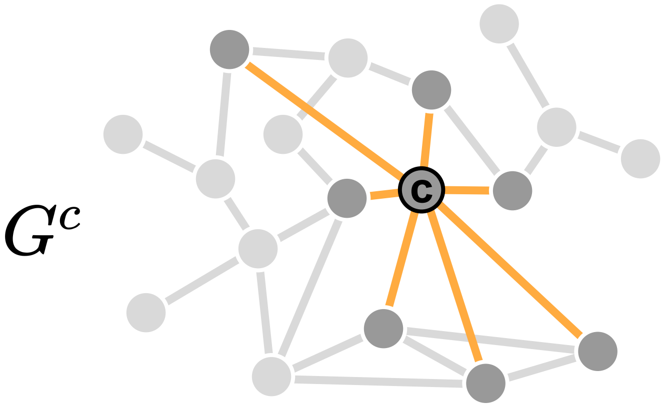

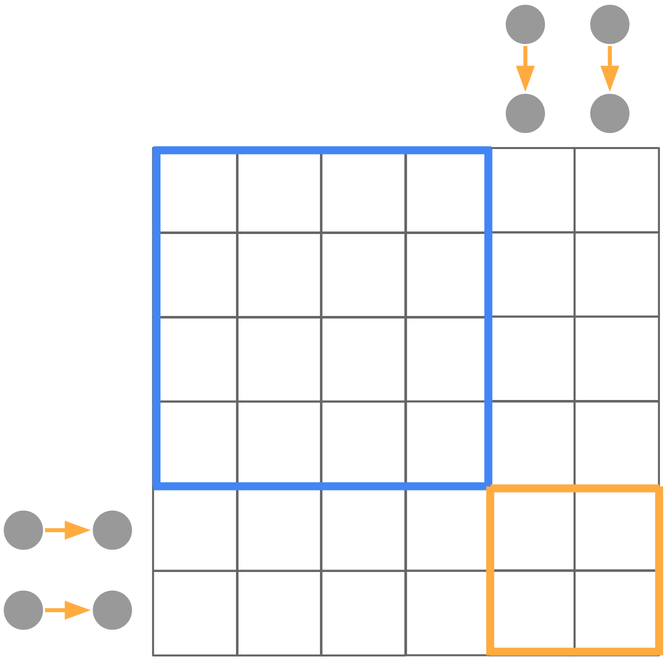

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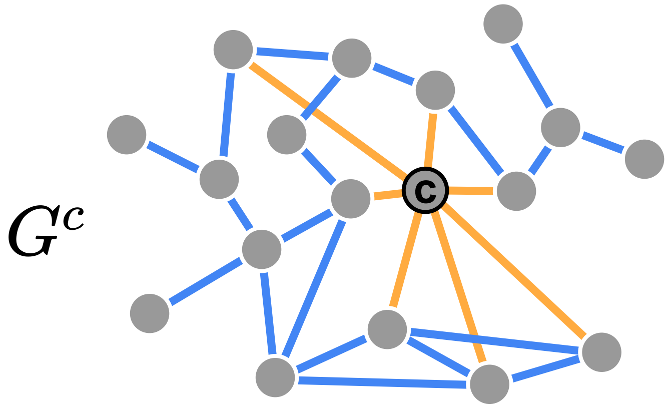
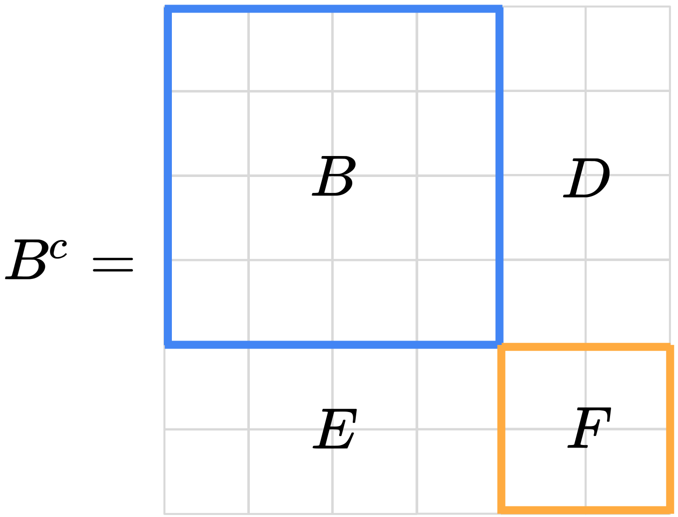
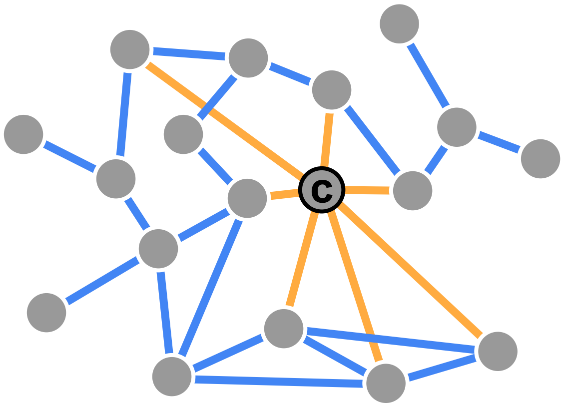


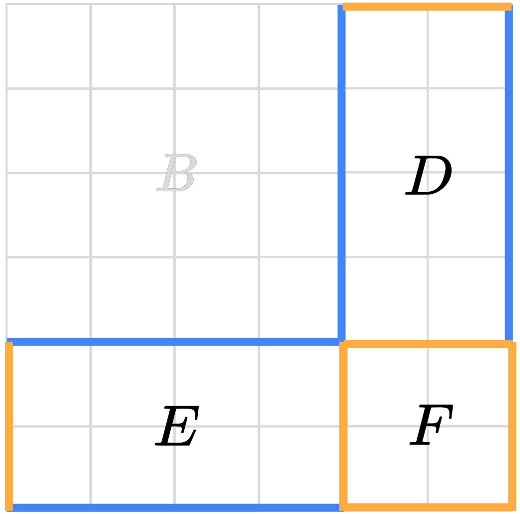


The X Matrix

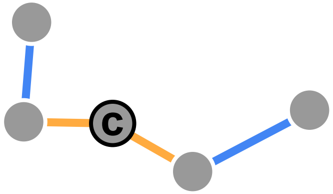
G^c



$B^c =$

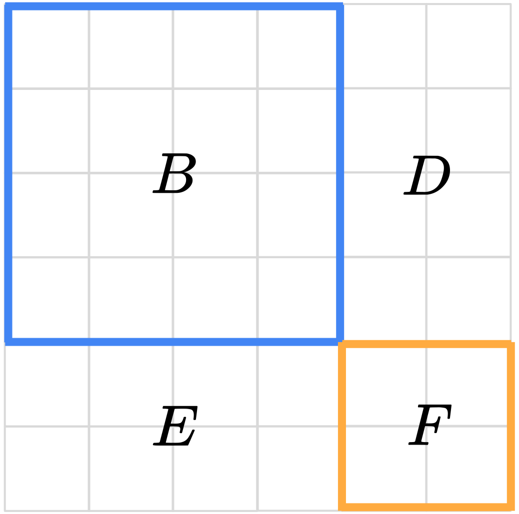


$X = DFE$



Solving for eigenvalues

$B^c =$



$$\det\left(B^c - tI\right) = 0$$

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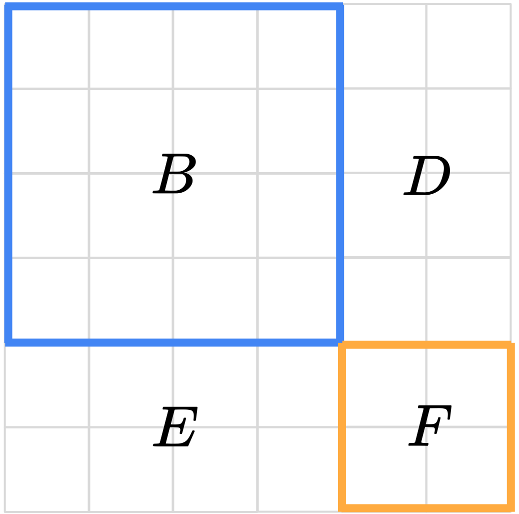

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determinant of block matrices

Solving for eigenvalues

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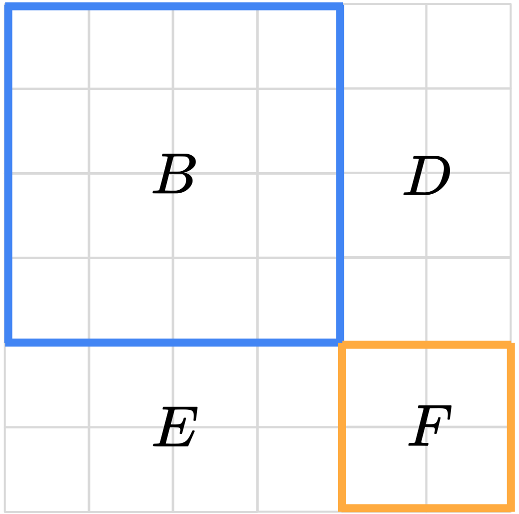

$$\det\left(B^c - tI\right) = 0$$

determinant of block matrices

$$\det\left(B^c - tI\right) = t^{2d}\det\left(B - tI\right)\det\left(I + \frac{YX}{t^2}\right)$$

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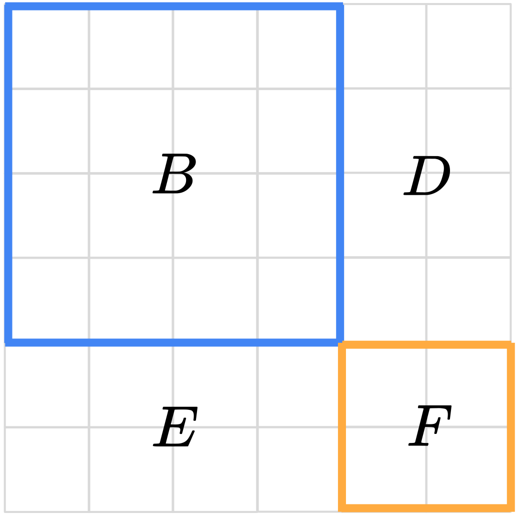

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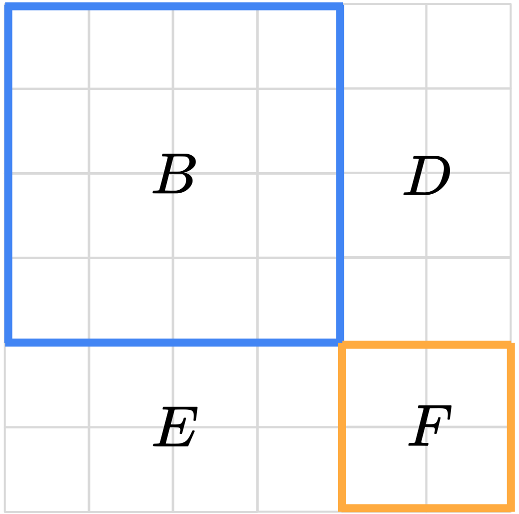

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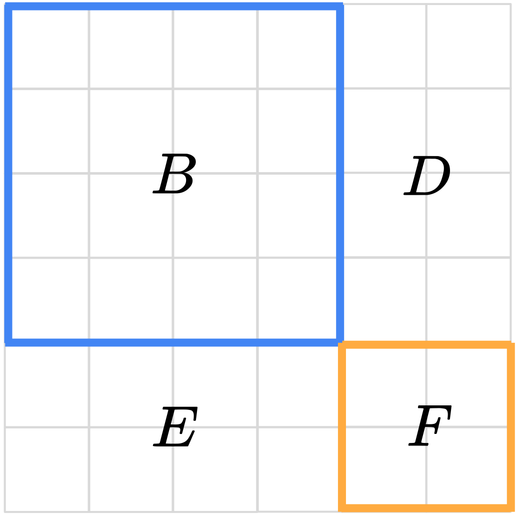

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linearize

Solving for eigenvalues

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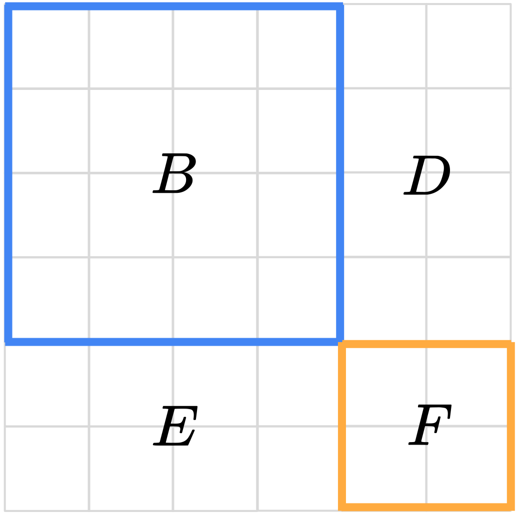

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$$\det\left(I + \frac{YX}{t^2}\right) = 1 + \frac{1}{t^2}Tr\left(YX\right) + \ldots$$

Solving for eigenvalues

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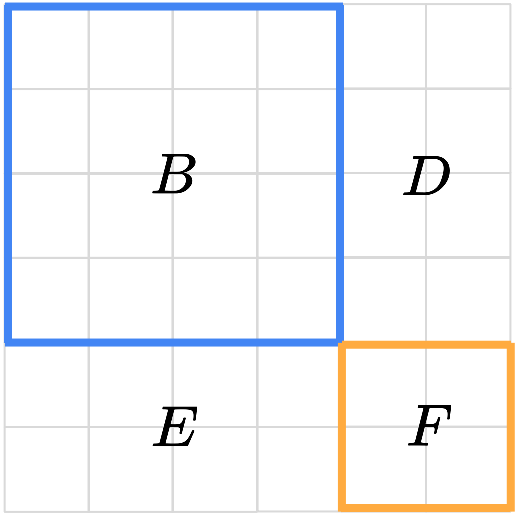

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u_i, v_i = **right and left eigenvectors**

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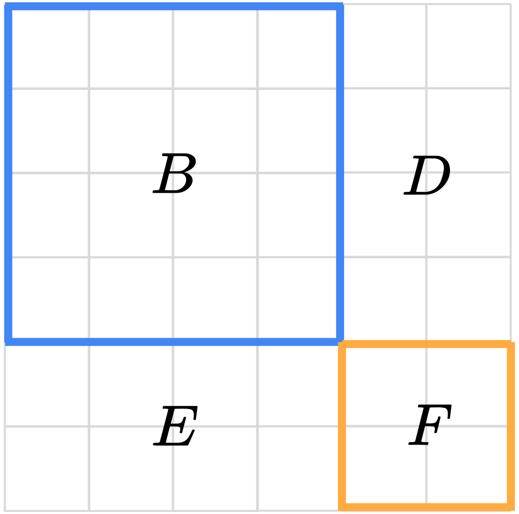

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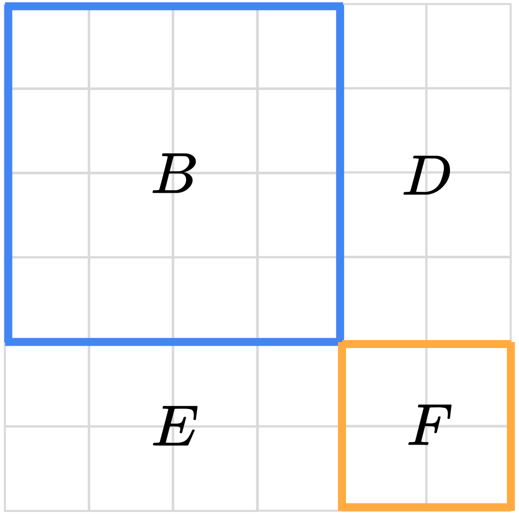

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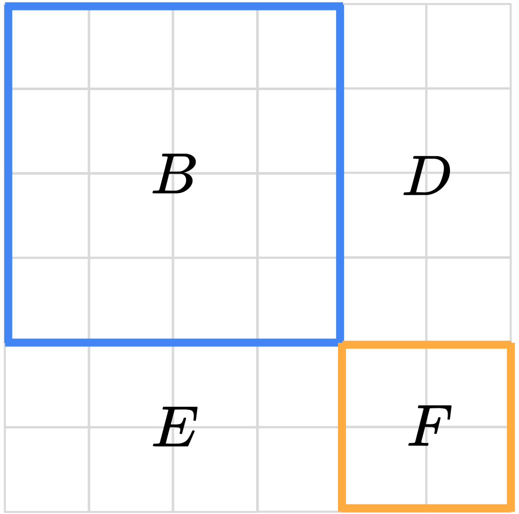

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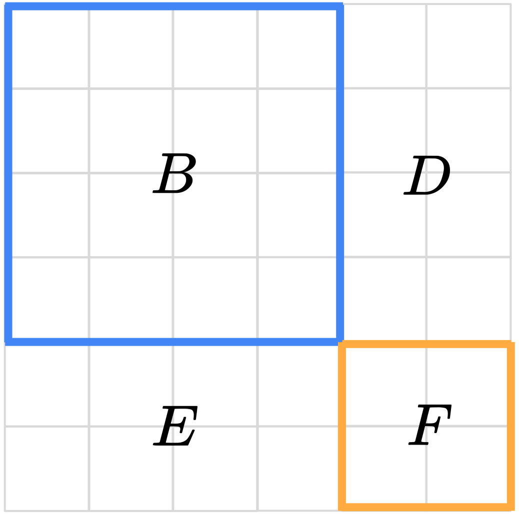
$$t^2(t - \lambda_1) + v_1^T X u_1 = 0$$

XNB Centrality

$B^c =$



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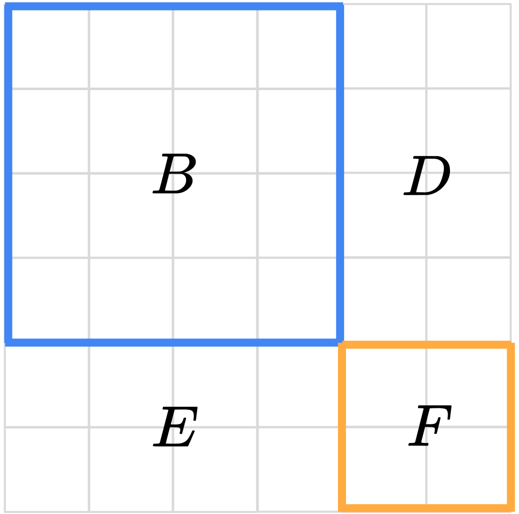

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X-non-backtracking centrality (**XNB**)

XNB Centrality

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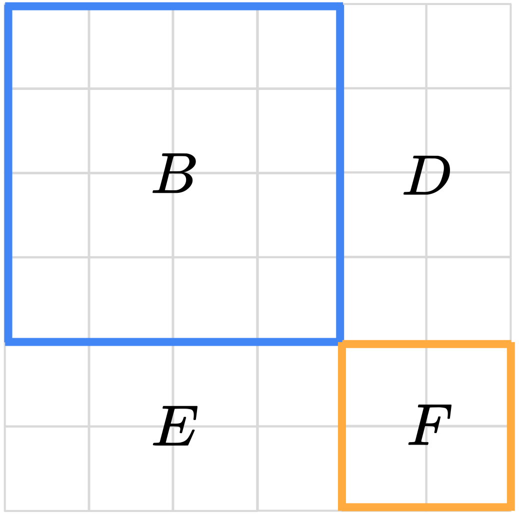

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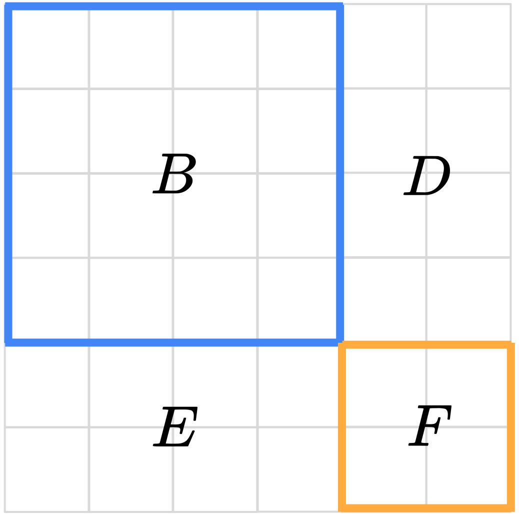
X-non-backtracking centrality
(**XNB**)

$$v_1^T X u_1 \leq \mathbf{1}^T X \mathbf{1}\,(1 + \dots)$$

X-deg Centrality

$B^c =$



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X-non-backtracking centrality (**XNB**)

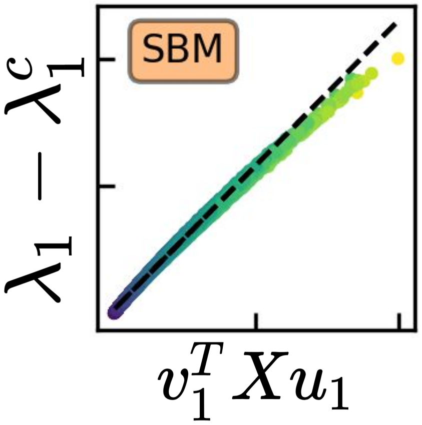
$$v_1^T X u_1 \leq \boxed{\mathbf{1}^T X \mathbf{1}} (1 + \dots)$$

X-degree centrality (**X-deg**)

What happens to the leading eigenvalue of the **non-backtracking** matrix when a node is removed from the graph?

It **decreases** by a quantity that is **correlated** to $v_1^T X u_1$

XNB and the true change in eigenvalue



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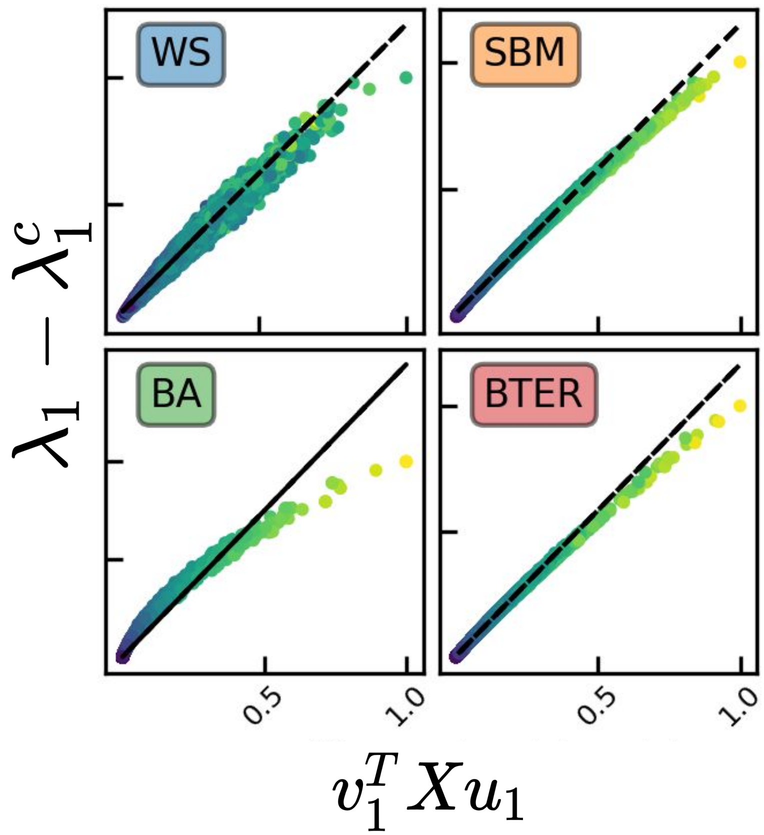

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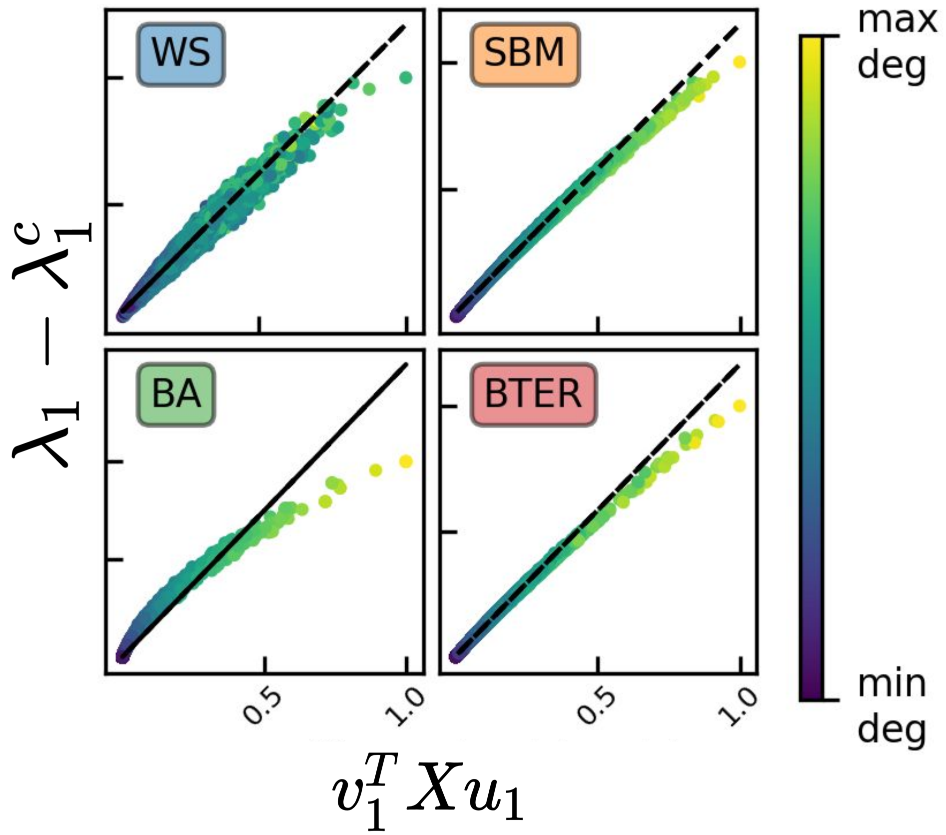

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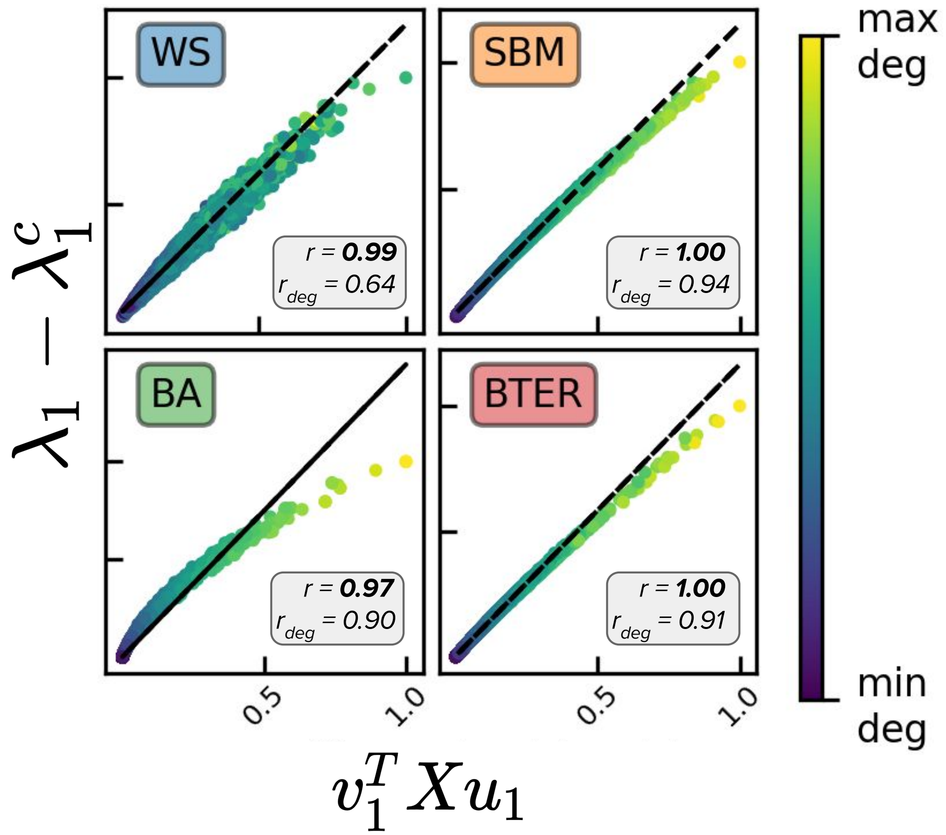

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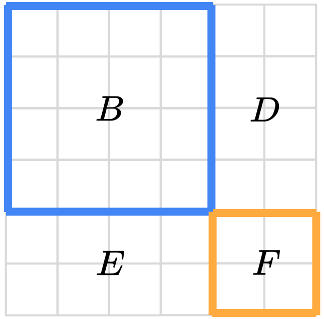
What happens to the leading eigenvalue of the **non-backtracking** matrix when a node is removed from the graph?

Use this knowledge to define a **centrality measure** for **node immunization**.

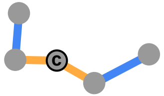
XNB for different target nodes

1. Choose a target node **c**

$B^c =$



$X_c = DFE$



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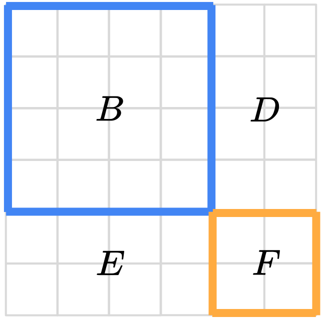

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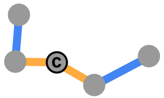

2. Compute u_1, v_1, and XNB

$$XNB(c) = v_1^T X_c u_1$$

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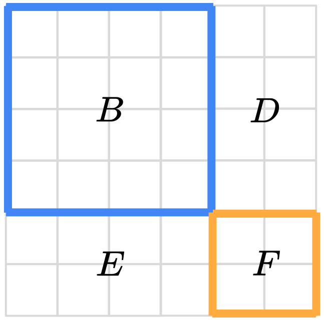

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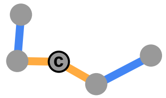

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3. Alternative way

$$XNB(c) = \left(\sum_i a_{ci} v_1^i\right)^2 - \sum_i a_{ci} \left(v_1^i\right)^2$$

v_1^i is the NB-centrality

X-deg for different target nodes

1. Choose a target node **c**

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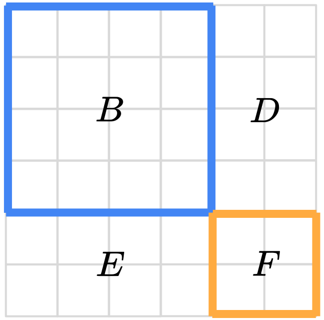
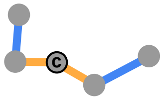


$X_c = DFE$



2. Compute u_1, v_1, and XNB

$$XNB(c) = v_1^T X_c u_1$$

$$Xdeg(c) = 1^T X_c 1$$

3. Alternative way

$$XNB(c) = \left(\sum_i a_{ci} v_1^i\right)^2 - \sum_i a_{ci} \left(v_1^i\right)^2$$

v_1^i is the NB-centrality

X-deg for different target nodes

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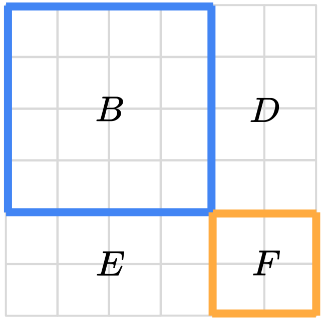

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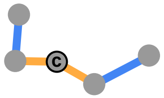

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$$Xdeg(c) = \left(\sum_i a_{ci} d_i'\right)^2 - \sum_i a_{ci} \left(d_i'\right)^2$$

$$d_i' = deg(i) - 1$$

"excess degree"

X-deg for different target nodes

1. Choose a target node **c**

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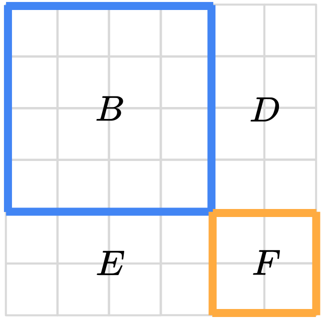

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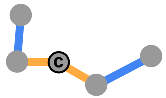

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Node immunization

Node immunization

Table 1: Average percentage eigen-drop (larger is better)

Node immunization

	1%
BA	2%
	3%
	1%
BTER	2%
	3%
	1%
SBM	2%
	3%
	1%
WS	2%
	3%

Table 1: Average percentage eigen-drop (larger is better) on synthetic graphs after removing 1%, 2%, and 3%

Node immunization

NS: Chen Chen et al. TKDE, 28 (1):113–126 (2016).
CI: Morone & Makse. Nature, 524(7563):65 (2015).

		degree	NS	CI	Xdeg	NB	XNB
BA	1%						
	2%						
	3%						
BTER	1%						
	2%						
	3%						
SBM	1%						
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WS	1%						
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		degree	NS	CI	Xdeg	NB	XNB
BA	1%	62.76	61.44	62.88	62.90	62.92	62.91
	2%	68.84	66.94	68.97	68.99	69.01	69.01
	3%	72.42	70.09	72.56	72.57	72.59	72.59
BTER	1%	6.28	6.40	6.41	6.45	6.46	6.46
	2%	10.60	10.72	10.80	10.85	10.86	10.86
	3%	14.31	14.40	14.55	14.61	14.63	14.63
SBM	1%	3.31	3.41	3.40	3.43	3.44	3.44
	2%	6.00	6.16	6.19	6.23	6.25	6.25
	3%	8.52	8.66	8.76	8.80	8.82	8.82
WS	1%	1.41	1.17	1.50	1.52	1.63	1.63
	2%	2.52	2.09	2.97	2.98	3.11	3.11
	3%	3.66	2.94	4.41	4.41	4.57	4.58

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Table 1: Average percentage eigen-drop (larger is better) on synthetic graphs after removing 1%, 2%, and 3%

Case study

AS-1
AS-2
Social-Slashdot
Social-Twitter
Transport-California
Transport-Sydney
Web-NotreDame

Case study

AS-1
AS-2
Social-Slashdot
Social-Twitter
Transport-California
Transport-Sydney
Web-NotreDame

Table 2: Average percentage eigen-drop on real networks (larger is better) when removing $p = 1, 10,$ or 100 nodes.

Case study

	$p = 1$		
	degree	CI	Xdeg
AS-1	0.74	0.74	**2.35**
AS-2	2.02	2.02	**4.00**
Social-Slashdot	0.95	**1.02**	**1.02**
Social-Twitter	**2.18**	**2.18**	1.98
Transport-California	0.00	0.00	**0.65**
Transport-Sydney	**0.00**	**0.00**	**0.00**
Web-NotreDame	**9.34**	**9.34**	**9.34**

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Web-NotreDame	**9.34**	**9.34**	**9.34**

Table 2: Average percentage eigen-drop on real networks (larger is better) when removing $p = 1, 10,$ or 100 nodes.

Case study

	$p = 1$			$p = 10$		
	degree	CI	Xdeg	degree	CI	Xdeg
AS-1	0.74	0.74	**2.35**	6.70	13.51	**15.43**
AS-2	2.02	2.02	**4.00**	17.09	22.36	**28.17**
Social-Slashdot	0.95	**1.02**	**1.02**	4.63	6.06	**6.94**
Social-Twitter	**2.18**	**2.18**	1.98	13.21	**13.97**	13.68
Transport-California	0.00	0.00	**0.65**	**2.65**	0.65	**2.65**
Transport-Sydney	**0.00**	**0.00**	**0.00**	0.00	0.00	**6.50**
Web-NotreDame	**9.34**	**9.34**	**9.34**	12.10	**13.79**	**13.79**

Table 2: Average percentage eigen-drop on real networks (larger is better) when removing $p = 1, 10,$ or 100 nodes.

Case study

	$p = 1$			$p = 10$			$p = 100$		
	degree	CI	Xdeg	degree	CI	Xdeg	degree	CI	Xdeg
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Social-Twitter	**2.18**	**2.18**	1.98	13.21	**13.97**	13.68	41.10	42.88	**43.39**
Transport-California	0.00	0.00	**0.65**	**2.65**	0.65	**2.65**	5.09	5.09	**7.80**
Transport-Sydney	**0.00**	**0.00**	**0.00**	0.00	0.00	**6.50**	0.00	7.37	**9.49**
Web-NotreDame	**9.34**	**9.34**	**9.34**	12.10	**13.79**	**13.79**	14.37	14.37	**19.22**

Table 2: Average percentage eigen-drop on real networks (larger is better) when removing $p = 1, 10,$ or 100 nodes.

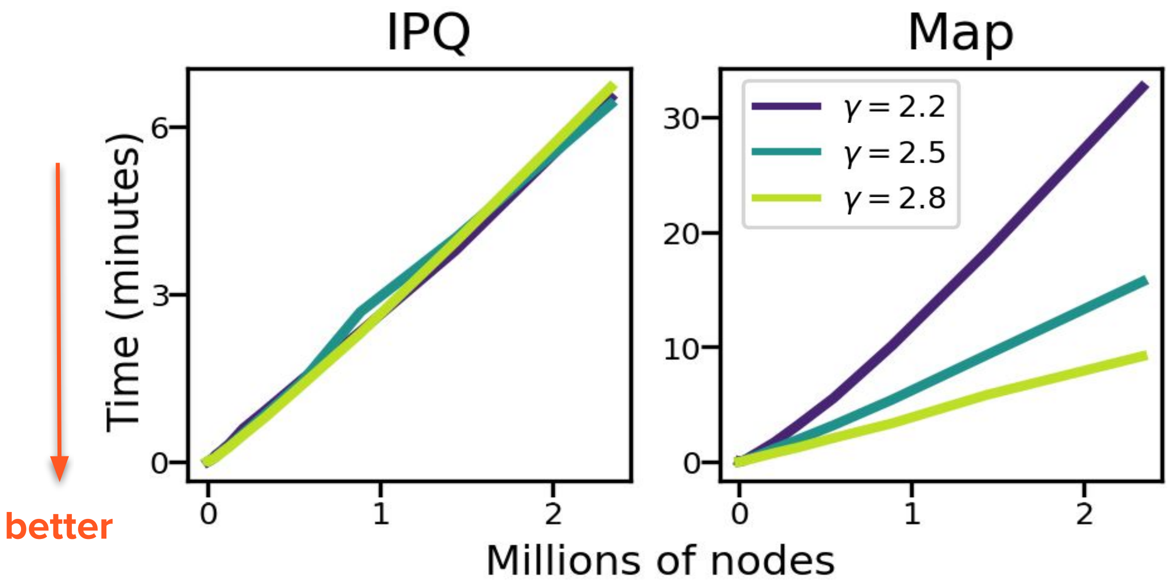
Algorithm: Immunization with XNB

Input: graph G, integer p
Output: removed, an ordered list of nodes

```
removed ← ∅
XNB [i] ← XCent(G,i) for each node i
while length(removed) < p do
    node ← max_i XNB [i]
    foreach i in G.neighbors[node] do
        G.neighbors[i].remove(node)
    foreach i in G.neighbors[node] do
        foreach j in G.neighbors[i] do
            XNB [j] ← XCent(G,i)
    G.neighbors[node] ← ∅
    removed.append(node)
return removed
```

This algorithm can be implemented using one of two data structures: **an indexed priority queue (IPQ), or a hash table (a.k.a. dictionary, Map)**. Each version is more efficient on **different types of networks.**

Algorithm: Scalability



Immunization on graphs with **heterogeneous degree distribution**
Real graphs typically have $2 < \gamma < 3$.

Future research

Why care about the non-backtracking matrix and its **eigenvalues**?

What happens to the leading eigenvalue of the **non-backtracking** matrix when a node is removed from the graph?

Use this knowledge to define a **centrality measure** for **node immunization**.

Cool, now what?

X-centrality and variance

$$Xdeg(c) = \left(\sum_i a_{ci} d'_i\right)^2 - \sum_i a_{ci} \left(d'_i\right)^2$$

$$d'_i = deg(i) - 1$$

"excess degree"

X-centrality and variance

$$Xdeg(c) = \left(\sum_i a_{ci} d_i'\right)^2 - \sum_i a_{ci} \left(d_i'\right)^2$$

$$-Var = \left(\frac{1}{\deg(c)} \sum_i a_{ci} d_i'\right)^2 - \frac{1}{\deg(c)} \sum_i a_{ci} \left(d_i'\right)^2$$

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$$CI(c) = d_c' \sum_i a_{ci}\, \boxed{d_i'}$$

$d_i' = deg(i) - 1$

"excess degree"

Question 1

$$Xdeg(c) = \left(\sum_i a_{ci} d'_i\right)^2 - \sum_i a_{ci} \left(d'_i\right)^2$$

$$-Var = \left(\frac{1}{\deg(c)} \sum_i a_{ci} d'_i\right)^2 - \frac{1}{\deg(c)} \sum_i a_{ci} \left(d'_i\right)^2$$

$$CI(c) = d'_c \sum_i a_{ci} d'_i$$

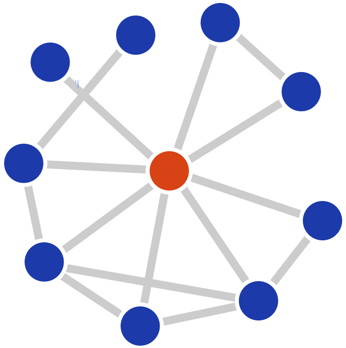
Future work:

- What about **higher** moments?

$d'_i = deg(i) - 1$

"excess degree"

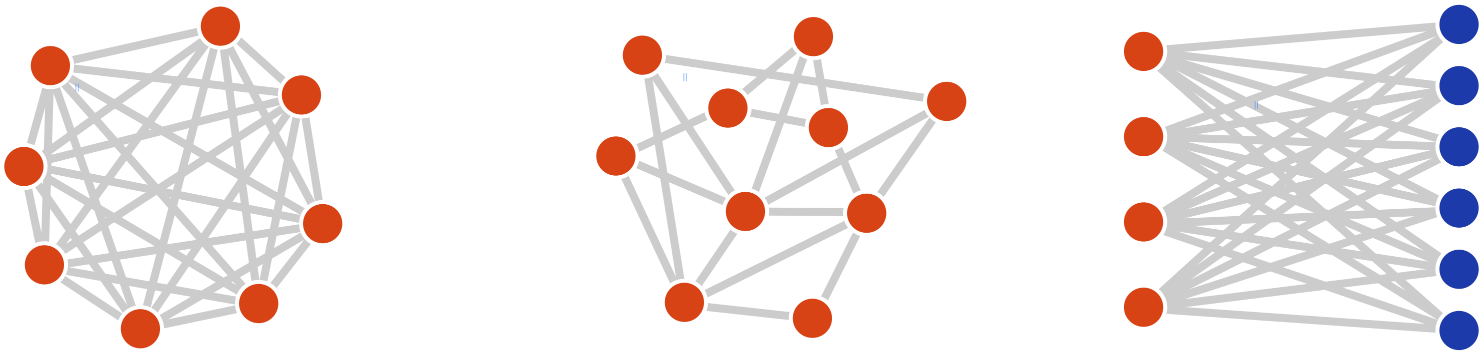
X-centrality and localization



The **adjacency** eigenvector is disproportionately **localized** on the **red node**.

Martin, et al. *Physical review E* 90.5 (2014): 052808.

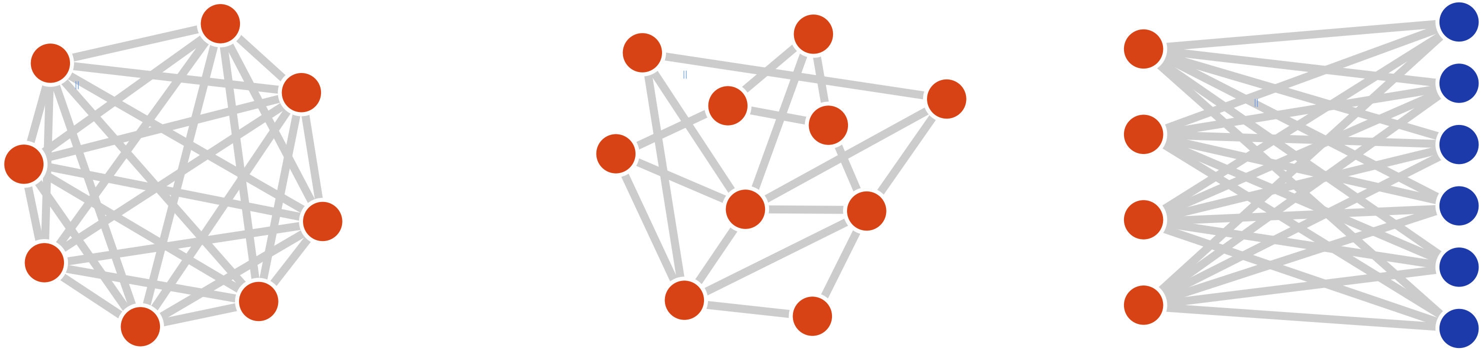
X-centrality and localization



The **non-backtracking** eigenvector is disproportionately **localized** on the **red nodes**.

Martin, et al. *Physical review E* 90.5 (2014): 052808.
Pastor-Satorras & Castellano. *Preprint* arXiv:2005.03913 (2020).

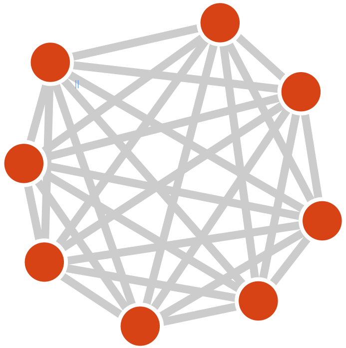
Question 2

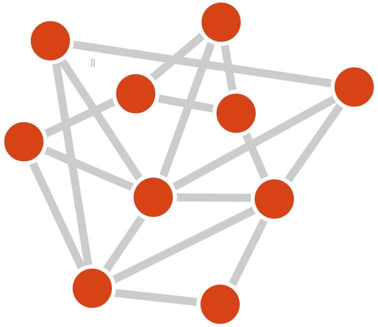


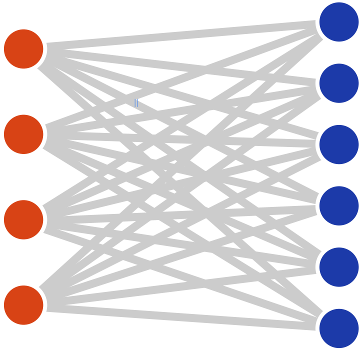
The non-backtracking eigenvector is disproportionately localized on the red nodes.

The degrees of the **neighbors** of the **red nodes** have **low variance** – thus the red nodes have **high X-deg**.

Question 2

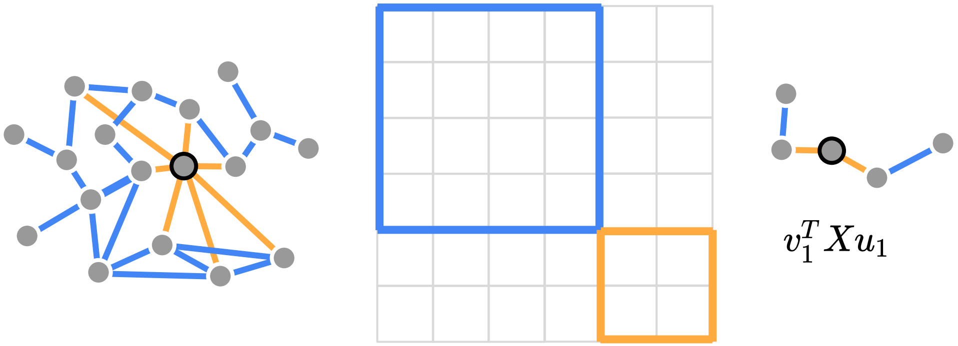






Generate a graph with a subset of nodes with **high X-degree**. Find a **localized eigenvector**?

Gracias!

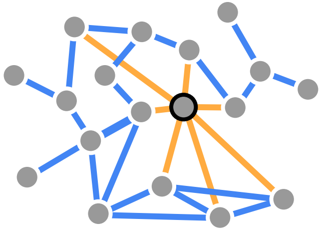


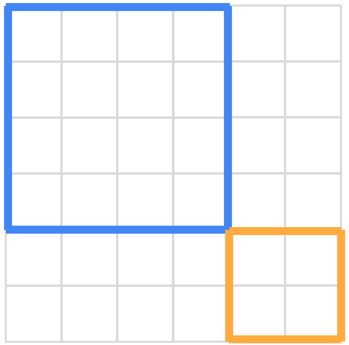
- The leading eigenvalue decreases by an amount correlated to **X-NB**.
- **New techniques** to analyze the non-backtracking matrix.
- Using X-centrality is **slightly** better in general, **largely** better in some cases.
- Connections with moments of neighbors' degree distribution, localization, etc.

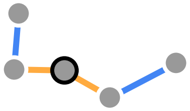
Paper: https://arxiv.org/abs/2002.12309

Code: https://github.com/leotrs/inbox

Gracias!







$v_1^T X u_1$

Kevin S. Chan, ARL

Hanghang Tong, UIUC

Tina Eliassi-Rad, NEU

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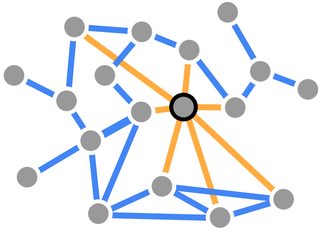

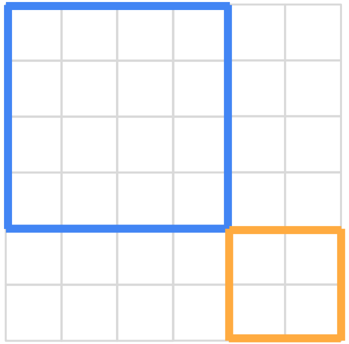

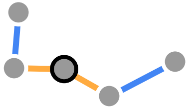

$v_1^T X u_1$

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Currently on the **job market** as a **postdoc** or **assistant professor** at the intersection of network science, computer science, and mathematics. Please get in touch!

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